NOTES

Analysis of the Packing Stage in Injection Molding of Disk Cavities

Injection molding is one of the most important polymer processing techniques. The process consists of three separate stages; filling, packing, and cooling. Usually, theoretical analyses[1-10] of the injection-molding process are concentrated on filling and cooling stages. Not much work, either theoretically or experimentally, has been done with regard to the packing stage. Kamal and Kenig[1-3] theoretically analyzed this stage assuming that the material movement into the cavity is proportional to the difference between the injection pressure and the average pressure within the cavity. A more detailed and complex mathematical analysis has been developed by Kamal et al.[11,12] Chung et al.[13] extended their analysis employing different initial and boundary conditions as well as a numerical method.

Most analyses[11-13] employed a linear relationship between pressure and density for the compressibility of molten polymers. However, the pressure–volume–temperature relationship for molten polymers generally does not follow a linear relation. In this note, we attempt to simulate the pressure build up of the packing stage using a generalized Newtonian fluid whose compressibility is considered to obey the Spencer–Gilmore equation of state.[14] In addition, a detailed examination of back flow in this stage will be given.

ANALYSIS

In the packing stage analysis of a thin disk cavity, the following assumptions are introduced:

(1) The fluid motion within the cavity is isothermal 2-dimensional radial flow of Newtonian fluid with pressure independent viscosity.

(2) Inertial forces, body forces, and viscoelastic effects are negligible.

(3) The compressible behavior of the melt obeys the Spencer–Gilmore equation of state:

$$(P + W)(1/\rho - 1/\rho_0) = R_c T \tag{1}$$

In the packing stage, the melt behaviors may be assumed to be Newtonian since the deformation rate is small.[12,13] The normal stresses in the dynamic equation may be neglected since the pressure loss in the thin disc cavity is mainly caused by the shear stress.[1-3,13] Temperature changes may be neglected because of the short duration of this stage (fraction of a second).

Then the relevant governing equations of continuity and motion become

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u) = 0 \tag{2}$$

$$\frac{\partial P}{\partial r} = \eta \frac{\partial^2 u}{\partial z^2} \tag{3}$$

Equation (3) can be easily solved to yield

$$u(r,z) = \frac{1}{2\eta}\left(-\frac{\partial P}{\partial r}\right)(h^2 - z^2) \tag{4}$$

In order to obtain the pressure buildup in the cavity, one can solve for the density buildup since they are interchangeable through eq. (1). Substituting P from eq. (1) into eq. (4) gives

$$u(r,z) = \frac{1}{2\eta}\frac{R_c T \rho_0^2}{(\rho_0 - \rho)^2}\left(-\frac{\partial \rho}{\partial r}\right)(h^2 - z^2) \tag{5}$$

Substituting eq. (5) into eq. (2) yields

$$\frac{\partial \rho}{\partial t} - \frac{(h^2 - z^2)\rho_0^2 R_c T}{2\eta}\left[\frac{\rho}{r(\rho_0 - \rho)^2}\frac{\partial \rho}{\partial r} + \frac{\rho_0 + \rho}{(\rho_0 - \rho)^3}\left(\frac{\partial \rho}{\partial r}\right)^2 + \frac{\rho}{(\rho_0 - \rho)^2}\frac{\partial^2 \rho}{\partial r^2}\right] = 0 \tag{6}$$

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The initial and the boundary conditions are as follows:

$$t = 0, \quad \rho = F_1(r) \tag{7}$$

$$t > 0, r \leq R_0, \quad \rho = F_2(t) \quad \text{or} \quad P = F_2'(t) \tag{8}$$

$$t > 0, r = R, \quad \frac{\partial \rho}{\partial r} = 0 \tag{9}$$

The initial condition results from the density distribution which exists at the end of the filling stage. Using the quasisteady state assumption and the lubrication approximation[3,6] for a power-law fluid, the following equation for the pressure distribution at the end of the filling stage can be derived:

$$P(r) = \frac{Amn}{1-n}(r^{1-n} - R^{1-n}) \tag{10}$$

where

$$A = -\frac{1}{n}\left(\frac{1+2n}{4\pi n h^{2+1/n}} Q\right)^n \tag{11}$$

and m and n are rheological parameters defined by

$$\tau = m\left(-\frac{du}{dz}\right)^n \tag{12}$$

Once the initial pressure distribution is known, the initial density distribution $F_1(r)$ can be easily obtained. $F_2'(t)$ describes a measurable pressure profile at the gate. The last boundary condition denotes no material flow at the side wall of the mold cavity.

RESULTS AND DISCUSSION

Equation (6) is a nonlinear partial differential equation of density and can be solved by a numerical scheme. Once the density distribution is calculated, the pressure profile can be easily obtained by using eq. (1). The polymers used in this analysis are a low density polyethylene and an amorphous polystyrene. Table I summarizes their compressibility and reheological parameters as well as mold geometry and operational condition. In this condition, polystyrene has higher viscosity than polyethylene at the low shear rate region. The packing pressure profiles $F_2'(t)$ were obtained from previous publications.[1–3]

The calculated pressure distributions at the center plane ($z = 0$) during packing for these two resins are shown in Figures 1 and 2. The normalized pressure is defined by ratio of the pressure P to the initial pressure P_0 at the cavity center. In the case of no backflow, the transitional pressure behavior

Table I
Material Properties and Molding Conditions

Material	Polystyrene	Polyethylene
n	0.368	0.594
m	252,800	30,320
η (P)	58,980	11,910
W (psi)	27,000	47,600
ρ_0 (g/cm^3)	1.2165	1.1429
R_c (psi·cm^3/g·°K)	11.6	43
T (°K)	450	420
R (cm)	10	10
R_0 (cm)	0.25	0.25
h (cm)	0.2	0.2
Q (cm^3/s)	50	50
P_0 (psi)	1195	460
$F_2'(t)$ (psi)	$P_0 + 10{,}000t$, $t < 0.2$	$P_0 + 20{,}000t$, $t < 0.1$
	2000, $t \geq 0.2$	2000, $t \geq 0.1$

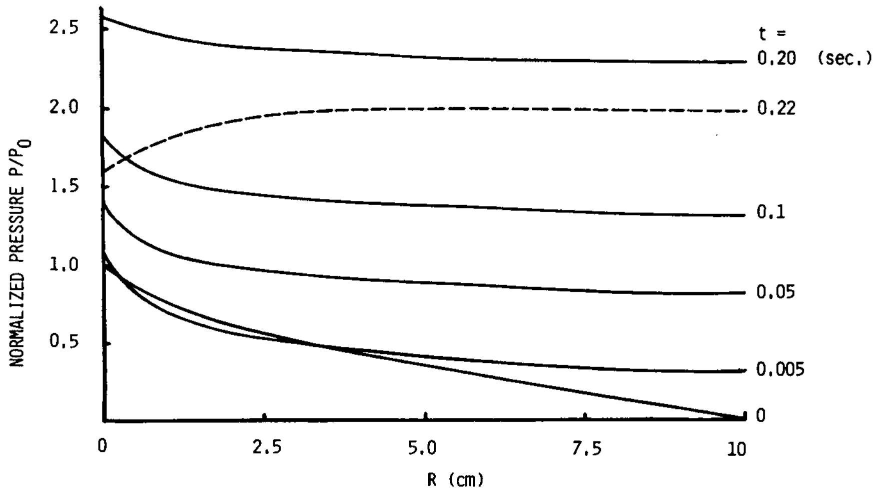

Fig. 1. Pressure buildup in the disk cavity for polystyrene.

for both polystyrene and polyethylene are quite similar. The pressure gradient is seen to be highest near the gate but becomes essentially uniform at a distance from the gate. One significant difference between these two polymers, however, is the rate of pressure rise: that of polyethylene is much faster than that of polystyrene. This phenomenon is consistent with experimental data reported in the literature[1-3] with the same material parameters. This difference comes from the difference in viscosity and compressibility. An additional interesting feature of both figures is that pressure near the gate decreases at the beginning of packing and then increases as a function of time. This is due to the effect of radial flow. The material flowing into these regions is less than the material moving out. In other words, the first term in the bracket of eq. (6) is greater than the addition of the last two terms. This phenomenon disappears rapidly whenever the pressure distribution becomes more uniform.

The dotted line in each figure shows the effect of backflow on the pressure profile. Backflow may be considered as a pressure flow caused by lower pressure at the head of the injection screw.[15] In actual operation, the screw retracts in order to fill the melt in the front chamber of the barrel just after the end of packing stage. The pressure developed in the front chamber or near the nozzle is

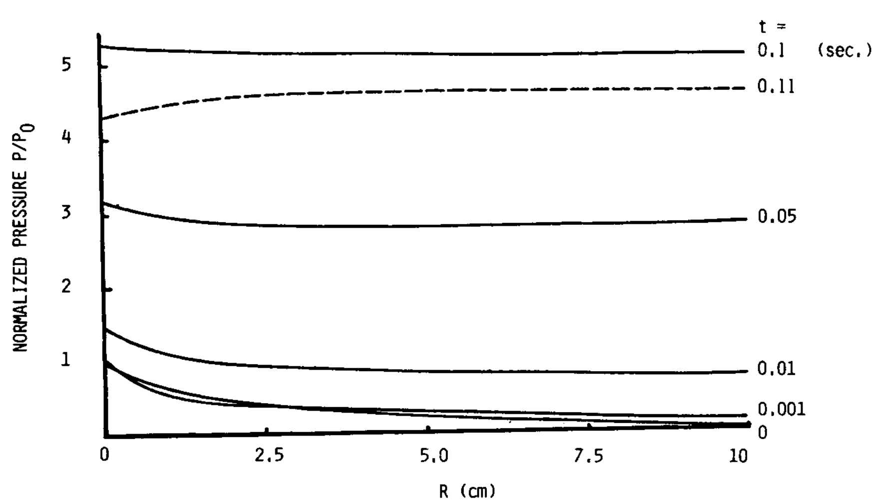

Fig. 2. Pressure buildup in the disk cavity for polyethylene.

therefore controlled by the screw rotational speed, operational temperature, as well as the nozzle and screw geometries. An increase in rotational speed generally results in an increase in melt pressure. However, this pressure is always lower than the gate pressure provided by the packing stage. As soon as the screw retracts, backflow occurs immediately and reduces the gate pressure. This sudden pressure drop may be simplified as a unit-step change as described in Table I. Figures 1 and 2 indicate that backflow generally causes a negative effect on the uniformity of pressure profile. Depending upon the extent of backflow, the pressure at the gate can become the lowest within the cavity. Therefore, this gate area generally has the largest shrinkage and the lowest mechanical properties. In order to reduce the backflow, the gate and runner should solidify as quickly as possible by incorporating proper designs (such as fast cooling rate, thin gate, or shutoff nozzle, etc.). On the other hand, since the backflow arises from the pressure difference between the gate and nozzle, reducing packing pressure or increasing nozzle pressure will ease the backflow and improve the quality of molded parts.

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APPENDIX: NOMENCLATURE

h	half thickness of mold cavity (cm)
m	rheological constant defined in eq. (12)
n	power law index
P	pressure (psi)
P_0	pressure at the cavity center at the end of filling (psi)
Q	volume flow rate (cm^3/cm)
R	radius of mold cavity (cm)
R_c	constant
R_0	radius of gate (cm)
r	radial coordinate
t	time (s)
W	constant (psi)
z	thickness direction
ρ	density (g/cm^3)
ρ_0	constant (g/cm^3)
η	Newtonian viscosity (P)
τ	shear stress

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